P425/1

PURE MATHEMATICS

Paper 1

Sept. 2022

3 hours

POST MOCK EXAMINATIONS 2022

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be shown clearly.*

Begin each answer on a fresh page.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

TURN OVER

SECTION A (40 MARKS)

*Answer **all** questions in this section.*

1. Solve the simultaneous equations: $\begin{aligned} 2x + 3y + 4z &= 8 \\ 3x - 2y - 3z &= -2. \\ 5x + 4y + 2z &= 3 \end{aligned}$ *(05 marks)*

2. Given that the polynomial $f(x) = x^3 - 3x^2 - 9x + k$ has a repeated root, find the possible values of k. *(05 marks)*

3. Use small changes to find $\sqrt{35.3}$ correct to four significant figures.

   (05 marks)

4. Solve for θ: $4sin\theta - 3cos\theta = 2$ for $0^0 \leq \theta \leq 360^0$. *(05 marks)*

5. Show that the lines l_1 and l_2, with vector equations $\boldsymbol{r} = \boldsymbol{k} + \mu(\boldsymbol{i} - \boldsymbol{j} - 3\boldsymbol{k})$ and $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{j} + \lambda(3\boldsymbol{j} + 5\boldsymbol{k})$ respectively, intersect, and find the coordinates of the point of intersection. *(05 marks)*

6. Evaluate: $\int_0^1 4x(3x^2 - 1)^3\, dx$. *(05 marks)*

7. Find the volume generated when the area between the curve $y = x^2$ and the line $y = x$ is rotated about the x-axis through 360^0. *(05 marks)*

8. If P is the point $(t, t^2 - 4)$ and Q is the point (2,0), find the equation of locus of R the midpoint of PQ as t varies. Describe the locus fully.

   (05 marks)

SECTION B: (60 MARKS)

*Answer **only five** questions from this section.*

9. (a) Assuming that x is very small that terms in x^3 can be neglected, find a quadratic approximation to $\sqrt{\frac{1-x}{1+2x}}$. State the range of values of x for which the expansion is valid. *(06 marks)*

   (b) Solve the equation: $\sqrt{5x-25} - \sqrt{x-1} = 2$. *(06 marks)*

10. (a) Given that $z = x + yi$ and $\frac{z-4}{z+3i}$ is purely real, find the cartesian equation of locus of z. *(06 marks)*

    (b) Solve for z: $(\bar{z} + 2z)\bar{z} = 5 + 2\bar{z}$ where $\bar{z}$ is a complex conjugate of $z = a + bi$ where a and b are non-zero real numbers. *(06 marks)*

11. (a) Evaluate: $\int_0^{\pi/2} cos3xcos2xdx$ *(04 marks)*

    (b) Express $\frac{11x+12}{(2x+3)(x^2-x-6)}$ in partial fractions. Hence find $\int \frac{11x+12}{(2x+3)(x^2-x-6)}\ dx$. *(08 marks)*

12. (a) Find the cartesian equation of the plane through the points A(1,0,-2) and B(3,-1,1) which is parallel to the line with vector equation $\boldsymbol{r} = 3\boldsymbol{i} + (2\lambda - 1)\boldsymbol{j} + (5-\lambda)\boldsymbol{k}$**.** Hence find the coordinates of the point of intersection of this plane and the line $\boldsymbol{r} = t\boldsymbol{i} + (5-t)\boldsymbol{j} + (2t-7)\boldsymbol{k}$**.** *(08 marks)*

    (b) Find the vector equation of the line through P(2,-1,3) and R(1,0,2). Verify that (-1,2,0) lies on the line. *(04 marks)*

13.(a) Given that $y = \frac{(x-1)^2 e^{4x}}{(x+1)^2}$, show that $\frac{dy}{dx} = \frac{4x^2(x-1)e^{4x}}{(x+1)^3}$. *(06 marks)*

(b) A champagne glass is in the shape of an inverted cone of depth 9cm and radius 3cm. Champagne is poured into the glass at the rate of 2π cm^3s^{-1}. Find the rate at which the depth of champagne in the glass is increasing 4 seconds after pouring has commenced. *(06 marks)*

14.(a) A chord of the parabola $y^2 = 4x$ given by $P({t_1}^2, 2t_1)$ and $Q({t_2}^2, 2t_2)$ subtends a right angle at the vertex.

Show that $t_1 t_2 = -4$ .Hence find the locus of the midpoint of the chord PQ. *(06 marks)*

(b) Find the equation of the circle that touches the line $y = x$ at the point A(3,3) and passes through the point B(5,9). *(06 marks)*

15.(a) Solve the equation: $sec\theta - 3cos\theta = sin\theta$ for $-180^0 \leq \theta \leq 180^0$.

(05 marks)

(b) Show that $\frac{cos3A}{cosA} + \frac{sin3A}{sinA} = 4cos2A$ *(04 marks)*

(c) Find without using mathematical tables or a calculator the value of $\frac{1+tan75^0}{1-tan75^0}$ in surd form. *(03 marks)*

16.Solve the following differential equations

(a) $\frac{dy}{dx} - 2ycotx = x^2 cosecx.$ *(06 marks)*

(b) $(1+x)\frac{dy}{dx} = 1 - cos2y$ given $y(0) = \frac{\pi}{4}$. *(06 marks)*

GOOD LUCK